选择题

1.C 2.D 3.B 4.C 5.B 6.D 7.A 8.C 9.D 10.D 11.A 12.A

填空题

13、7

14、240

15、

$$\frac{\sqrt{2}}{3}\pi$$

16、②③

17.(1) $a_1=3 \quad a_{n+1}=3a_n-4n$

$a_2=3a_1-4=5$

$a_3=3\cdot a_2-4\cdot 2=7$ 猜想 $a_n=2n+1$

① 当 $n=1$ 时 $a_1=2\times1+1=3$ 成立

② 假设 $n=k$ 时 $a_k=2k+1$ 成立 当 $n=k+1$ 时

$a_{k+1}=3a_k-4k=3(2k+1)-4k=2k+3=2(k+1)+1$

综上所述 $a_n=2n+1$

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(2) 由 $b_n=2^n a_n=2^n(2n+1)$

$S_n=3\cdot2+5\cdot2^2+7\cdot2^3+\cdots+(2n-1)2^{n-1}+(2n+1)2^n$ ①

$2S_n=3\cdot2^2+5\cdot2^3+\cdots+(2n-1)2^n+(2n+1)2^{n+1}$ ②

$-S_n=6+2\cdot2^2+2\cdot2^3+\cdots+2\cdot2^n-(2n+1)2^{n+1}$

$=6+\dfrac{8(1-2^{n-1})}{1-2}-(2n+1)2^{n+1}$

$=-2-(2n-1)2^{n+1}$

$\therefore S_n=(2n-1)2^{n+1}+2$

18 (1) 设空气质量等级为X，则

$P(X=1)=\frac{2+16+25}{100}=0.43$

$P(X=2)=\frac{5+10+12}{100}=0.27$

$P(X=3)=\frac{6+7+8}{100}=0.21$

$P(X=4)=\frac{7+2+0}{100}=0.09$

(2) 由题意 设锻炼人次为Y 有Y可取100, 300, 500

$P(Y=100)=\frac{2+5+6+7}{100}=0.2$

$P(Y=300)=\frac{16+10+7+2}{100}=0.35$

$P(Y=500)=\frac{25+12+8+0}{100}=0.45$

∴ 所求 $=EY=0.2\times100+0.35\times300+0.45\times500=20+105+225=350$ 人次

(3) 由题

	人次≤400	人次>400
空气质量好	33	37
〃 不好	22	8

$K^2=\frac{100(33\times8-22\times37)^2}{70\times30\times55\times45}=\frac{121\times4(12-37)^2}{7\times3\times55\times45}=\frac{11\times4\times25}{7\times3\times9}$

$=\frac{1100}{189}\doteq5.82$ 又 $P(K^2\geq3.841)=0.05$

∴ 有95%的把握认为一天中 ______

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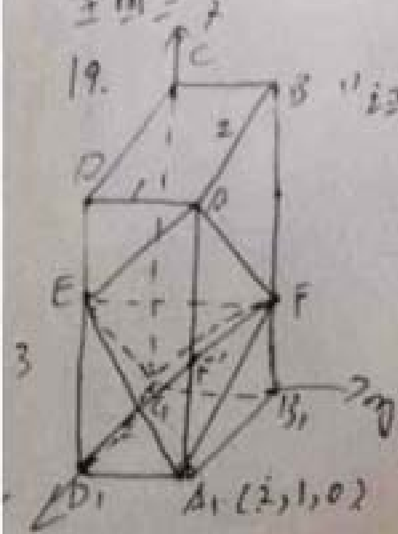
19.



1) 证明 取在 AA' 上取点 $A_1F' = B_1F$.

连接 FF', D_1F', C_1F.

则 $C_1F \parallel= D_1F'$

又 ∵ $AF' \parallel= ED_1$

∴ $F'D_1 \parallel= AE$

∴ $C_1F \parallel= AE$

∴ 四边形 AEC_1F 为平行四边形

∴ 点 C_1 在平面 AEF 内.

2). 取 C_1 为坐标原点 C_1D_1 为 x, C_1B_1 为 y, CC_1 为 z

则 $A_1(2,1,0)$. $E(2,0,2)$ $F(0,1,1)$. $A(2,1,3)$

$\vec{AE}=(0,-1,-1)$. $\vec{AF}=(-2,0,-2)$

设平面 AEF 法向量 $\vec{n}(x_1,y_1,z_1)$

$\vec{n}\cdot\vec{AE} = -y_1 - z_1 = 0$ 令 $z_1=1$. 则 $y_1=-1$

$\vec{n}\cdot\vec{AF} = -2x_1 - 2z_1 = 0$ $x_1=-1$

∴ $\vec{n}=(-1,-1,1)$.

设平面 A_1EF 法向量 $\vec{m}(x_2,y_2,z_2)$

$\vec{A_1E}=(0,-1,2)$. $\vec{A_1F}=(-2,0,1)$

$\vec{m}\cdot\vec{A_1E} = -y_2+2z_2=0$. 令 $z_2=1$. $y_2=2$.

$\vec{m}\cdot\vec{A_1F} = -2x_2+z_2=0$ $x_2=\frac{1}{2}$

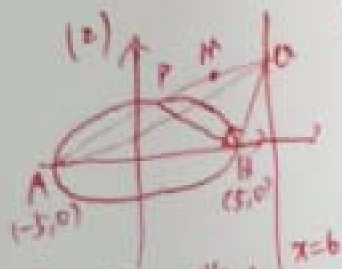
∴ $\vec{m}=(\frac{1}{2},2,1)$.

$\Rightarrow \sin\theta = |\cos\langle \vec{n}, \vec{PB}\rangle| = \dfrac{\ }{\sqrt{3}\cdot\sqrt{2\lambda^2+1}}$

$= \dfrac{1}{\sqrt{3}\sqrt{\frac{2}{\lambda^2}-\frac{2}{\lambda}+1}} \le \dfrac{1}{\sqrt{3}\sqrt{\frac{1}{2}}} = \dfrac{\sqrt{6}}{3}$

20. (1) $e = \dfrac{\sqrt{25-m^2}}{5} = \dfrac{\sqrt{5}}{4} \Rightarrow m^2 = \dfrac{25}{16} \Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{\frac{25}{16}} = 1$

(2)



设 $Q(6, y_0)$. ⇒ ~~$k_{BQ} = \frac{y_0}{1}$~~

$\Rightarrow k_{BQ} = y_0$, $BP \perp BQ$ $\Rightarrow k_{BP} = -\dfrac{1}{y_0}$

$\Rightarrow BP: y - 0 = -\dfrac{1}{y_0}(x-5) \Rightarrow \begin{cases} x = 5 - y_0 y \\ x^2 + 16y^2 = 25 \end{cases} \Rightarrow$

$\Rightarrow (16+y_0^2)y^2 - 10y_0 y = 0$

$\Rightarrow y_P + y_B = \dfrac{10y_0}{16+y_0^2} \Rightarrow y_P = \dfrac{10y_0}{16+y_0^2}$, $x_P = \dfrac{80-5y_0^2}{16+y_0^2}$

$\vec{AP} = \left(\dfrac{160}{16+y_0^2}, \dfrac{10y_0}{16+y_0^2}\right)$

$\vec{AQ} = (11, y_0)$

$\Rightarrow S_{\triangle APQ} = \dfrac{1}{2}\left|\dfrac{160y_0}{16+y_0^2} - \dfrac{110y_0}{16+y_0^2}\right| = \dfrac{50}{2}\dfrac{|y_0|}{16+y_0^2} = \dfrac{50}{2}\dfrac{1}{\frac{16}{|y_0|}+|y_0|}$

$= \dfrac{25}{17}$

$M\left(\dfrac{176+y_0^2}{2(16+y_0^2)}, \dfrac{26y_0+y_0^3}{2(16+y_0^2)}\right)$

由 $k_{AM}\cdot k_{PQ} = -\dfrac{26y_0+y_0^3}{16-9y_0^2}\cdot\dfrac{6y_0+y_0^3}{11y_0^2+16} = -1$

得 $|y_0| = 1$

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$$\cos\theta = \frac{\vec{m}\cdot\vec{n}}{|\vec{m}||\vec{n}|}$$

$$= \frac{-\frac{1}{2}}{\sqrt{\frac{3}{4}\times 3}} = \frac{-1}{\sqrt{7}}$$

$$= -\frac{\sqrt{7}}{7}$$

$$\sin\theta = \sqrt{1-\frac{1}{7}} = \sqrt{\frac{6}{7}}$$

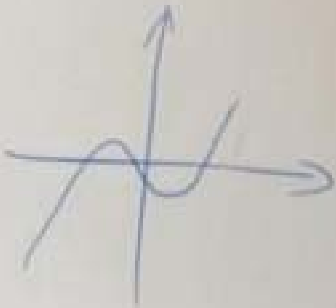
$$= \frac{\sqrt{42}}{7}$$

21.(1) $f'(x)=3x^2+b$ $f'(\frac{1}{2})=3\times(\frac{1}{2})^2+b=\frac{3}{4}+b$

由题意 $f'(\frac{1}{2})=0$ ∴ $\frac{3}{4}+b=0$ ∴ $b=-\frac{3}{4}$

(2) $f(x)=x^3-\frac{3}{4}x+c$ $f'(x)=3x^2-\frac{3}{4}=3(x+\frac{1}{2})(x-\frac{1}{2})$

x	$(-\infty,-\frac{1}{2})$	$-\frac{1}{2}$	$(-\frac{1}{2},\frac{1}{2})$	$\frac{1}{2}$	$(\frac{1}{2},+\infty)$
$f'(x)$	+	0	−	0	+
$f(x)$	↗	极大值	↘	极小值	↗



$f(-\frac{1}{2})=(-\frac{1}{2})^3-\frac{3}{4}(-\frac{1}{2})+c=-\frac{1}{8}+\frac{3}{8}+c=\frac{1}{4}+c$

$f(\frac{1}{2})=(\frac{1}{2})^3-\frac{3}{4}\times\frac{1}{2}+c=\frac{1}{8}-\frac{3}{8}+c=c-\frac{1}{4}$

当 $c-\frac{1}{4}>0$ 时 即 $c>\frac{1}{4}$ 时 $f(x)=0$ 仅有一个零点 故结论成立

当 $\frac{1}{4}+c<0$ 时 即 $c<-\frac{1}{4}$ 时   〃   〃

当 $c=\frac{1}{4}$ 时 $f(\frac{1}{2})=0$. $f(x)=x^3-\frac{3}{4}x+\frac{1}{4}=(x^3-\frac{1}{2}x^2)+(\frac{1}{2}x^2-\frac{1}{4}x)-(\frac{1}{2}x-\frac{1}{4})$

$=(x-\frac{1}{2})(x^2+\frac{1}{2}x-\frac{1}{2})=(x-\frac{1}{2})^2(x+1)$ 三个根为 $\frac{1}{2}$, -1 合题

当 $c=-\frac{1}{4}$ 时 $f(-\frac{1}{2})=0$ $f(x)=x^3-\frac{3}{4}x-\frac{1}{4}=(x+\frac{1}{2})^2(x-1)$ 二根为 $-\frac{1}{2}$, 1 合题

当 $-\frac{1}{4}<c<\frac{1}{4}$ 时 $f(1)=1^3-\frac{3}{4}\times1+c=\frac{1}{4}+c>0$ $f(-\frac{1}{2})=\frac{1}{4}+c>0$

$f(-1)=(-1)^3-\frac{3}{4}(-1)+c=-\frac{1}{4}+c<0$ $f(\frac{1}{2})=c-\frac{1}{4}<0$

又由f单调性知 $f(x)=0$ 有三根 $x_1<x_2<x_3$

且 $-1<x_1<-\frac{1}{2}<x_2<\frac{1}{2}<x_3<1$

综上 ______

22. (1) 令 $x=0$ 得 $2-t-t^2=0 \Leftrightarrow t^2+t-2=0 \Leftrightarrow (t-1)(t+2)=0$

又 $t\neq 1$ ∴ $t=-2$

当 $t=-2$ 时 $y=2-3(-2)+(-2)^2=2+6+4=12$ 即C过 $(0,12)$

令 $y=0$ 得 $2-3t+t^2=0 \Leftrightarrow (t-1)(t-2)=0$ 又 $t\neq 1$ ∴ $t=2$

当 $t=2$ 时 $x=2-2-2^2=-4$ ∴ C过 $(-4,0)$

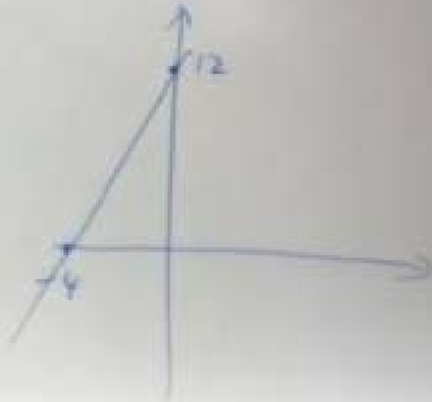
∴ $|AB|=\sqrt{(0-(-4))^2+(12-0)^2}=\sqrt{16+144}=4\sqrt{10}$

(2) $AB:\ \dfrac{x}{-4}+\dfrac{y}{12}=1$

即 $-3x+y=12$

$\Leftrightarrow -3\rho\cos\theta+\rho\sin\theta=12$

$\Leftrightarrow \rho=\dfrac{12}{\sin\theta-3\cos\theta}$



23 (1) $c=-a-b$ 由$abc=1$知a,b,c均不为0且有正数

不妨设 $c>0$ 则 $ab>0$

$\because (a+b)^2 \geq 4ab$ $\therefore (-c)^2 \geq 4ab \Rightarrow c^2 \geq 4ab > ab$

$ab+bc+ca = ab+c(a+b) = ab-c^2 < 0$

(2) 由a,b,c的对称性，不妨设 $a<b<c$ 由(1)知 $c>0$

由(1) $c^2 \geq 4ab = 4\cdot\frac{1}{c} \Rightarrow c^3 \geq 4 \Rightarrow c \geq \sqrt[3]{4}$

即 $\max\{a,b,c\} \geq \sqrt[3]{4}$